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Volume stabilization via α' corrections in type IIB theory with fluxes

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Abstract: We consider the type IIB string theory in the presence of various extra $7/\overline{7}$-brane pairs compactified on a warped Calabi-Yau threefold that admits a conifold singularity. We demonstrate that the volume modulus can be stabilized perturbatively at a non-supersymmetric AdS_4/dS_4 vacuum by the effective potential that includes the stringy $(\alpha')^3$ correction obtained by Becker et al. together with a combination of positive tension and anomalous negative tension terms generated by the additional 7-brane-antibrane pairs.

Keywords: D-branes, Superstring Vacua.

*Currently at DUMC.

Contents

1. Introduction

Moduli stabilization in sting theory using warped compactifications [1] with nontrivial fluxes has proven to be of great success [2 − 12]. In the case of type IIB warped compactifications on an orientifolded Calabi-Yau with background NS and RR three-form fluxes, all complex-structure moduli as well as the dilaton can be stabilized. Because of the "no-scale" structure of the resulting supergravity potential, the Kähler moduli remain unfixed. It was argued that α' stringy corrections and non-perturbative effects may break the "no-scale" structure providing a mechanism to freeze the Kähler moduli [2]. It has subsequently been demonstrated that the non-perturbative contributions to the superpotential [13, 14] can indeed stabilize the volume modulus producing a supersymmetric AdS_4 vacuum [17]. Moreover, by adding extra sources such as anti-D3-branes one can brake supersymmetry and tune the fluxes to lift the AdS_4 ground state to obtain a metastable dS_4 [17]. Getting de Sitter space from string theory has been a rather difficult problem. In particular, the "no-go" theorem [18, 19] states that a de Sitter solution cannot be obtained in string or M-theory if one only uses the low-energy supergravity action. Sigma model α' corrections, perturbative effects in g_s and inclusion of extended sources were expected to violate the "no-go" conditions which was successfully demonstrated by an explicit construction [17]. A different way to obtain a dS space solution in string theory was found earlier in [20, 21] where some non-geometric effects at the string scale were used. The scenario of Kachru, Kallosh, Linde and Trivedi (KKLT) [17] has been more succesfull since it incorporates many attractive model-building features such as branes moving in a warped geometry which may provide a possible solution to the hierarchy problem in a context of a brane world model. Yet, more generic models can probably be constructed at the string scale so it becomes a matter of time to see which scenario prevails in the end. Variations of the original KKLT model have also been found [22, 23]. For constructions of de Sitter solutions in Heterotic string theory see [24, 25]. In a recent work, the authors of [26] suggested to modify the

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KKLT proposal by incorporating stringy corrections to the Kähler potential [27] to obtain non-supersymmetric vacua including dS_4. A new alternative to the KKLT construction of de Sitter vacua appeared recenlty in [28] where flux compactifications on products of Riemann surfaces are considered and the leading effects stabilizing the moduli are perturbative. For good reviews see [29 – 31]. Many inflationary models motivated by the KKLT scenario have been proposed [32 – 37, 43, 38 – 44]. In a recent paper [43] the authors discuss some generic problems that exist in the simplest version of the KKLT motivated inflationary models. In particular, they point out at the restriction that relates the inflationary Hubble scale and SUSY breaking scale $H \lesssim m_{3/2}$ which implies that in the simplest version of KKLT based inflation, high energy scale of SUSY breaking is favored. They then propose a more general setup based on racetrack type superpotentials where this restriction can be avoided altogether. Apart from the above generic problem, a different type of obstacle appears in the brane-antibrane inflation realized in warped geometry [32]. This problem is related to the superpotential volume stabilization mechanism used in the KKLT scenario. More specifically, the authors of [32] argue that for a generic functional form of the superpotential, the inflaton potential gets modified in a way that makes it too steep for inflation. One possible resolution to this problem briefly discussed in [32] would be Kähler volume stabilization mechanism. This method would allow one to stabilize the volume modulus directly without generating a large inflaton mass. On the other hand, in the case of superpotential volume stabilization volume modulus is a component of a superfield which is stabilized. This is the key difference that makes Kähler volume stabilization compatible with the brane-antibrane inflation.

We consider a type IIB theory warped compactification with non-trivial NS and RR fluxes on a Calabi-Yau threefold $\mathcal{M}$ that admits a conifold singularity with various 7-branes wrapped on the four-cycle inside $\mathcal{M}$. Supersymmetry is broken by a non-zero $(0,3)$ component of the $G_{(3)}$ flux that generates a Gukov-Vafa-Witten [45] superpotential $W \neq 0$. In the F-theory picture this is a compactification on an elliptically-fibered Calabi-Yau fourfold X with a Calabi-Yau threefold $\mathcal{M}$ as the base of the fibration and the 7-branes embedded into the base at the special loci where the fiber T^2 degenerates. All complex-structure moduli are stabilized by a choice of the integer fluxes [2].

We show that the volume modulus as well as the dilaton can be stabilized perturbatively when we combine the stringy corrections to the "no-scale" potential obtained in [27] with the contributions induced by additional pairs of (p,q) $7/\overline{7}$-branes and $D7/\overline{D7}$-branes wrapped on the four-cycle.[1] Apart from the positive tension terms which dominate at large volume, the extra brane-antibrane pairs also generate an anomalous negative tension term proportional to the triple intersection numbers that has a subdominant large volume dependence. We also assume that the loci where the T^2 fiber degenerates are located far from the highly warped region so the warp factor scaling the potential energy due to the 7-branes is trivial and the volume dependence is not modified by the warping. When the Euler characteristic of the Calabi-Yau is negative $\chi(\mathcal{M}) < 0$ the perturbative

[1] Our construction is a hybrid that features a warped compactification geometry as in [17] but uses a perturbative mechanism for volume stabilization similar to [28].

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stringy correction yields a positive potential that dominates at small volume over the other contributions in the effective potential.

By discretely varying the integer fluxes, the Euler characteristic and the number of extra brane-antibrane pairs in the effective potential we obtain a "landscape" of metastable de Sitter, non-supersymmetric Minkowski and anti-de Sitter vacua where supersymmetry is broken at the Kaluza-Klein scale. Such flexibility in the choice of flux, topological and brane numbers implies that we can stabilize the volume at a large value and the string coupling g_s at a small value, providing theoretical control over the perturbative expansion in the low-energy effective field theory. Such control is necessary in the absence of low-energy supersymmetry as loop corrections at various orders modifying the moduli potential will appear [28]. In addition, in this regime, the non-perturbative corrections to the superpotential are exponentially suppressed and can be safely neglected.

2. Review of type IIB warped compactifications with fluxes

In type IIB compactifications on a Calabi-Yau threefold $\mathcal{M}$ with nontrivial NS and RR fluxes $H_{(3)}$ and $F_{(3)}$ [2, 3, 46] the fluxes induce warping of the background giving a warped product of Minkowski space and the Calabi-Yau:

$$ds^2 = e^{2A(y)}\eta_{\mu\nu}dx^\mu dx^\nu + e^{-2A(y)}g_{mn}dy^m dy^n \,, \tag{2.1}$$

where g_{mn} is the Calabi-Yau metric. To construct such solution we are forced to include extended local sources to cancel the D3-brane charge produced by the fluxes following from the Bianci identity

$$d\tilde{F}_{(5)} = H_{(3)} \wedge F_{(3)} \,. \tag{2.2}$$

In the language of F-theory compactified on an elliptically fibered Calabi-Yau fourfold X, to satisfy flux conservation (2.2) we can introduce various 7-branes sitting at the special loci in the base $\mathcal{M}$ where the elliptic fiber T^2 degenerates. The 7-branes wrap the four-cycles in $\mathcal{M}$ generating an anomalous negative D3-brane charge induced by the curvature of the four-cycles $Q_3^7 = -\chi(X)/24$, where $\chi(X)$ is the Euler characteristic of the fourfold. In this language, the varying profile of the type IIB axion-dilaton $\tau \equiv C_{(0)} + ie^{-\phi}$ corresponds to the complex structure of the elliptic fiber. In the limit of the type IIB Calabi-Yau orientifold, we can introduce a set of $N_{\rm O3}$ orientifold O3-planes which quotient the space by a discrete symmetry and carry a D3-brane charge $Q_3^{\rm O3} = -N_{\rm O3}/4$. The total D3-brane charge required to vanish is then

$$Q_3^{local} + \frac{1}{2\kappa_{10}^2 T_3}\int_{\mathcal{M}} H_{(3)} \wedge F_{(3)} = 0 \,, \tag{2.3}$$

where Q_3^{local} includes contributions from all local sources of the D3-brane charge: 7-branes or O3-planes, mobile D3 and $\overline{\rm D3}$-branes. The three-form fluxes induce a Gukov-Vafa-Witten (GVW) superpotential [45] for the compex-structure moduli

$$W = \int_{\mathcal{M}} \Omega \wedge G_{(3)} \,, \tag{2.4}$$

where $G_{(3)} \equiv F_{(3)} - \tau H_{(3)}$ and Ω is the holomorphic three-form of the Calabi-Yau. The Calabi-Yau threefold $\mathcal{M}$ in general has a large number of both complex-structure moduli z_a and Kähler moduli ρ_i as well as the axion-dilaton τ. The geometric moduli surviving the orientifold projection include $h^{1,1}_+$ even Kähler deformations and $h^{1,2}_-$ odd complex-structure deformations.[2] The axion-dilaton also survives the projection. In further analysis we will assume a large volume limit and later check it for consistency. Here we will use units where the four-dimensional Newton's constant $\kappa_4^2 = (8\pi G_N) = 1$. For the simple case of a single Kähler modulus ρ, where $\mathrm{Im}\rho = \hat{\sigma} = e^{4u-\phi}$ corresponds to the fluctuations of the overall volume $\mathcal{V} = e^{6u}$, the tree-level Kähler potential takes the following form:

$$K = -3\log\left(-i(\rho - \bar{\rho})\right) - \log\left(-i\int_{\mathcal{M}} \Omega \wedge \overline{\Omega}\right) - \log\left(-i(\tau - \bar{\tau})\right). \tag{2.5}$$

A more general case with several Kähler moduli and a nontrivial warp factor is considered in [11]. The standard $\mathcal{N} = 1$ supergravity potential is given by

$$V = e^K \left(\mathcal{G}^{A\bar{B}} D_A W \overline{D_{\bar{B}} W} - 3|W|^2\right), \tag{2.6}$$

where $D_A W = \partial_A W + W \partial_A K$ and $\mathcal{G}_{A\bar{B}} = \partial_A \partial_{\bar{B}} K$. The structure of the Kähler potential (2.5) is such that when we use it to compute the potential (2.6) the $-3|W|^2$ term cancels out and the potential reduces to the familiar "no-scale" form [2]

$$V = e^K \left(\mathcal{G}^{a\bar{b}} D_a W \overline{D_{\bar{b}} W} + \mathcal{G}^{\tau\bar{\tau}} D_\tau W \overline{D_{\bar{\tau}} W}\right). \tag{2.7}$$

Notice, that $A, \bar{B}$ in (2.6) run over all moduli, whereas in (2.7) the indices run only over the complex-structure moduli $z_a, \bar{z}_b$ and axion-dilaton τ. This implies that even after we freeze z_a's and τ, the Kähler modulus ρ remains unfixed. However, see [23] for an interesting way around this problem where nonzero local minima of the no-scale potential were found. One of the conditions to obtain a warped solution (2.1) is the requirement that the three-form $G_{(3)}$ must be imaginary self-dual [2]:

$$*_6 G_{(3)} = i G_{(3)}, \tag{2.8}$$

which is equivalent to the following requirements

$$\begin{aligned} D_\tau W &= 0, \\ D_a W &= 0. \end{aligned} \tag{2.9}$$

The Klebanov-Strassler (KS) solution [47] is an example of such warped geometry where the six-dimensional internal space is a warped deformed conifold with a tip smoothed by an S^3. The fluxes satisfy the following quantization conditions

$$\frac{1}{(2\pi)^2\alpha'}\int_A F_{(3)} = M, \qquad \frac{1}{(2\pi)^2\alpha'}\int_B H_{(3)} = -K, \tag{2.10}$$

[2]See a recent work [46] on constructing the effective action for N=1 type IIB on Calabi-Yau orientifolds.

where A is a shrinking S^3 at the tip of the conifold and B is a dual cycle giving the following GVW superpotential [2]:

$$W = (2\pi)^2\alpha' \left[M\mathcal{G}(z) - K\tau z\right], \tag{2.11}$$

where z is a complex coordinate on the collapsing cycle defined by

$$z = \int_A \Omega\,, \tag{2.12}$$

and on the dual cycle

$$\int_B \Omega \equiv \mathcal{G}(z) = \frac{z}{2\pi i}\ln z + \text{holomorphic}\,. \tag{2.13}$$

Once the integer values M and K in (2.10) are fixed and condition (2.8) is satisfied, all complex structure moduli z_a are frozen. The dilaton can also be fixed if we turn on fluxes on another pair of three cycles A', B'. The warp factor has a minimum value at the bottom of the throat given by

$$e^{A_{\min}} \sim z^{1/3} \sim \exp\left(\frac{-2\pi K}{3Mg_s}\right), \tag{2.14}$$

creating a hierarchy of energy scales determined by the choice of the integer fluxes. In a generic case supersymmetry is broken by a non-vanishing $(0,3)$ part of $G_{(3)}$ that induces a constant non-zero value for the superpotential $W = W_0$. Requiring unbroken supersymmetry amounts to requiring that $D_\rho W = 0$, which implies that the superpotential $W = 0$ since it is independent of ρ. The authors of [17] proposed a mechanism to stabilize the Kähler modulus by including ρ-dependent non-perturbative corrections to the tree-level superpotential [13 – 16]:

$$W = W_0 + Ae^{ia\rho}\,. \tag{2.15}$$

In this way the "no-scale" structure of (2.7) gets broken resulting in a potential with a supersymmetric AdS_4 minimum where the volume modulus is stabilized at a large value. A further step was then taken to introduce anti-D3-branes that break supersymmetry and generate, due to their tension, a positive potential which dominates at large volume. By varying the number of anti-D3-branes and integer fluxes, the minimum in the effective potential can be lifted to obtain a discrete "landscape" of metastable de Sitter vacua [17].

3. Volume stabilization and de Sitter space

3.1 BBHL stringy corrections

The authors of [2] suggested that the "no-scale" structure of the potential (2.7) may be broken in the quantum theory by higher order stringy corrections. Such higher order $(\alpha')^3$ corrections to the potential were explicitly computed by Becker, Becker, Haack, and Louis (BBHL) [27] based on the previous work [48 – 50] in the context of a type IIB warped compactification with three-form fluxes. The new terms found in [27] originate from the correction of the Kähler moduli part of the Kähler potential which in string frame reads:

$$K = -2\log\left[2\mathcal{V}e^{-3\phi_0/2} + \xi\left(\frac{-i(\tau-\bar\tau)}{2}\right)^{3/2}\right] - \log\left(-i\int_{\mathcal{M}}\Omega\wedge\overline{\Omega}\right) - \log\left(-i(\tau-\bar\tau)\right), \tag{3.1}$$

while the flux induced superpotential (2.4) remains uncorrected [2, 51]. The term in the first line in (3.1) contains the dilaton-dependent $(\alpha')^3$ correction proportional to

$$\xi = -\frac{\chi(\mathcal{M})}{2}\zeta(3)\,, \tag{3.2}$$

where $\chi(\mathcal{M})$ is the Euler characteristic of the Calabi-Yau threefold and we used the convention of [27] to set $2\pi\alpha' = 1$. In their computation, the authors of [27] did not include the warp factor e^{4A}. It would be interesting to see if including it would change our further conclusions. For now we will assume that we consider a large volume limit so the warp factor can be ignored. The volume modulus $\mathcal{V}$ can be expressed as an implicit function of the volumes of the four-cycles σ_i in terms of the areas of the two-cycles v^k and in the large volume limit is given by:

$$\mathcal{V} = \int_{\mathcal{M}} J^3 = \frac{1}{6}k_{ij\,k}v^i v^j v^k\,, \qquad \sigma_i = \frac{1}{6}k_{ij\,k}v^j v^k\,, \tag{3.3}$$

where J is the Kähler form and $k_{ij\,k}$ are constant intersection numbers.[3] The physical volume of the Calabi-Yau

$$\mathrm{V}_{CY} \equiv \mathcal{V}(2\pi\alpha')^3 = \mathcal{V}\,. \tag{3.4}$$

In the four-dimensional Einstein frame the Kahler moduli are rescaled as

$$\hat{v}_i = v_i e^{-\phi_0/2}\,, \qquad \hat{\sigma}_i = \sigma_i e^{-\phi_0}\,, \tag{3.5}$$

so $\hat{\mathcal{V}} = \mathcal{V}e^{-3\phi_0/2}$. In order to keep using the same notation as in section 2, we will identify the complexified Kähler moduli as $\rho_i = \frac{1}{3}g_i + i\hat{\sigma}_i$, where g_i is the axion coming from the KK-reduction of the RR four-form. Including the $(\alpha')^3$ corrections in the Kähler potential breaks the "no-scale" structure resulting in the following form of the potential [27]:

$$\begin{aligned} V =&\; e^K\left(\mathcal{G}^{a\bar{b}}D_aW\overline{D_{\bar{b}}W} + \mathcal{G}^{\tau\bar{\tau}}D_\tau W\overline{D_{\bar{\tau}}W}\right) - 9\frac{\hat{\xi}\hat{\mathcal{V}}e^{-\phi_0}}{(\hat{\xi}-\hat{\mathcal{V}})(\hat{\xi}+2\hat{\mathcal{V}})}e^K\left(W\overline{D_{\bar{\tau}}W} + \overline{W}D_\tau W\right) - \\ &-3\hat{\xi}\,\frac{(\hat{\xi}^2 + 7\hat{\xi}\hat{\mathcal{V}} + \hat{\mathcal{V}}^2)}{(\hat{\xi}-\hat{\mathcal{V}})(\hat{\xi}+2\hat{\mathcal{V}})^2}e^K|W|^2\,, \end{aligned} \tag{3.6}$$

where $\hat{\xi} = \xi e^{-3\phi_0/2}$ and the components of the inverse metric necessary to compute (3.6) are:

$$\begin{aligned} \mathcal{G}^{\tau\bar{\tau}} &= e^{-2\phi_0}\frac{4\hat{\mathcal{V}}-\hat{\xi}}{\hat{\mathcal{V}}-\hat{\xi}}\,, \\ \mathcal{G}^{\tau\bar{\rho}_i} &= e^{-\phi_0}\frac{3\hat{\xi}}{\hat{\mathcal{V}}-\hat{\xi}}\hat{\sigma}_i\,, \\ \mathcal{G}^{\rho_i\bar{\rho}_j} &= -\frac{2}{9}(2\hat{\mathcal{V}}+\hat{\xi})k_{ij\,k}\hat{v}^k + \frac{4\hat{\mathcal{V}}-\hat{\xi}}{\hat{\mathcal{V}}-\hat{\xi}}\hat{\sigma}_i\hat{\sigma}_j\,. \end{aligned} \tag{3.7}$$

[3] The indices are raised with δ^{ij}.

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Once we fix the integer fluxes and impose conditions (2.9) necessary to obtain a warped solution (2.1), all complex-structure moduli z_a become fixed and potential (3.6) takes the following form:

$$V = 3\hat{\xi}\,\frac{(\hat{\xi}^2 + 7\hat{\xi}\hat{\mathcal{V}} + \hat{\mathcal{V}}^2)}{(\hat{\mathcal{V}} - \hat{\xi})(2\hat{\mathcal{V}} + \hat{\xi})^2} e^K |W|^2\,, \tag{3.8}$$

Notice that the potential is proportional to the square of the flux induced superpotential (2.4) and therefore vanishes unless supersymmetry is broken by the $(0,3)$ part of the three-form flux $G_{(3)}$.

3.2 Volume modulus stabilization

The authors of [27] rightly point out that the result in (3.8), containing all orders in α' beginning with $(\alpha')^3$, can only be trusted to order $(\alpha')^3$. In fact, to this leading order this correction to the "no-scale" potential is exact. Thus, keeping the leading order terms, we obtain the following potential to order $(\alpha')^3$:

$$V_s = \frac{3\hat{\xi}}{4\hat{\mathcal{V}}} e^{K^{(0)}} |W|^2 = -A\frac{\chi |W|^2}{2}\left(\frac{g_s^4}{\mathcal{V}^3}\right)\,, \tag{3.9}$$

where $K^{(0)}$ is the supergravity Kähler potential

$$K^{(0)} = \phi_0 - 2\log(\hat{\mathcal{V}}) - \log\left(-i\int_{\mathcal{M}} \Omega\wedge\overline{\Omega}\right) + \text{const.}\,, \tag{3.10}$$

and A is a numerical constant. In the four-dimensional Einstein frame this type of classical higher derivative correction should scale as g_s^2 in string coupling. Normally, this is indeed the case, since the superpotential W is linear in τ and $\mathrm{Im}(\tau) = 1/g_s$ so the factor of $|W|^2$ in (3.9) gives the expected scaling. However, in the case of a highly warped geometry at the bottom of the Klebanov-Strassler throat, the GVW superpotential (2.11) is effectively given by

$$W \sim (2\pi)^2\alpha' M\,. \tag{3.11}$$

Here we used the fact that the holomorphic part of $\mathcal{G}(0) \sim O(1)$ and the second term in (2.11) containing τ is multiplied by z and therefore is exponentially suppressed as we take $K/(g_s M) \gg 1$ to obtain a large hierarchy of scales in the highly warped region. Unfortunately, this type of potential by itself still cannot fix the volume modulus because of its runaway behaviour at large volume where it vanishes.[4]. However, recall that in our compactification scheme we had to include certain extended sources of the D3-brane charge in order to cancel the tadpole anomaly (2.3) In the F-theory language such objects were various 7-branes wrapped on the four-cycle inside the base $\mathcal{M}$, that induced an anomalous D3-brane charge $-\chi(X)/24$, where $\chi(X)$ is the Euler characteristic of the Calabi-Yau fourfold X. In case of low-energy supersymmetry, this anomalous negative contribution to the D3 charge is related to the negative tension by a BPS condition. The corresponding negative energy contribution is then cancelled by one of the positive terms due to the fluxes

[4] The authors of [26] combined the perturbative stringy $(\alpha')^3$ correction with non-perturbative contributions to the superpotential (2.15) to find non-supersymmetric AdS_4/dS_4 vacua.

making the total energy positive semi-definite [2]. We will therefore use the same strategy that was used by Silverstein[5] and Saltman in the recent work [28]. We will introduce n_{D7} additional pairs of $D7/\overline{D7}$-branes and n_7 extra pairs of (p,q) $7/\overline{7}$-branes wrapped on the four-cycle in $\mathcal{M}$ placed at the loci where the fiber T^2 degenerates. These pairs induce no net D3-brane charge so the Gauss law (2.3) is still satisfied. In fact, in a non-supersymmetric case considered here, a configuration that includes additional brane-antibrane pairs is more generic.[6] These extra local sources generate an anomalous negative D3-brane tension which can be tuned to a large value independently of the total D3-brane charge which gives us a lot of extra flexibility. In particular, in Einstein frame, these extra brane-antibrane pairs make the following total contribution to the effective potential [28]:

$$V_7 \sim -N_7\left(\frac{g_s^3}{\mathcal{V}^2}\right) + n_7\left(\frac{g_s^2}{\mathcal{V}^{4/3}}\right) + n_{D7}\left(\frac{g_s^3}{\mathcal{V}^{4/3}}\right), \tag{3.12}$$

where N_7 is an effective parameter given in terms of triple intersections of 7-branes and is proportional to qubic combinations of n_7 and n_{D7}. Here we have assumed that the points in the base $\mathcal{M}$ where the F-theory elliptic fiber degenerates are located far from the highly warped region. Therefore we have set the warp factor scaling the 7-brane tensions to unity $e^{4A} \approx 1$. Notice that the negative contribution is qubic in n_7 and n_{D7} and dominates at small volume over the other two terms. This large negative energy makes annihilation of brane-antibrane pairs energetically disfavored. One important step needs to be done before we combine contributions (3.9) and (3.12). Recall that in (3.9) the volume is measured in units of $(2\pi\alpha')^3$ as in (3.4) whereas in (3.12), the volumes are given in units of $(\alpha')^3$ which we will use in the total effective potential. Thus, going to the latter unit convention we obtain the following effective potential up to factors of order one

$$V_t = -\chi(2\pi)^{13}M^2\left(\frac{g_s^4}{\mathcal{V}^3}\right) - N_7\left(\frac{g_s^3}{\mathcal{V}^2}\right) + n_7\left(\frac{g_s^2}{\mathcal{V}^{4/3}}\right) + n_{D7}\left(\frac{g_s^3}{\mathcal{V}^{4/3}}\right), \tag{3.13}$$

where we also used (3.11) to substitute for W and set $\alpha' = 1$. Notice that if the Calabi-Yau manifold has a negative Euler characteristic $\chi(\mathcal{M}) < 0$, which is the case if the number of complex-structure deformations is larger than the number of Kähler deformations, the first term in (3.13) becomes positive. It dominates over the other terms at small volume and therefore the total effective potential features a minumum at a finite volume. By discretely varying the fluxes, the Euler characteristic and numbers of brane-antibrane pairs we obtain a "landscape" of metastable de Sitter, non-supersymmetric Minkowski and anti-de Sitter vacua. To find the minimum of (3.13) we can first take a derivative with respect to the string coupling and by setting the derivative to zero, we can find g_s in terms of $\mathcal{V}$. In fact, this is relatively easy since in the end it boils down to solving a simple quadratic equation that gives the following

$$g_s = -\frac{3N_7\mathcal{V} - 3n_{D7}\mathcal{V}^{5/3} + \Delta^{1/2}}{8\chi(2\pi)^{13}M^2}, \tag{3.14}$$

[5] I would like to thank Eva Silverstein for explaining some key points related to this part of the paper.

[6] In [28] the authors used n_7 additional sets of 24 (p,q) 7-branes whose contributions are known explicitly from F-theory on K3.

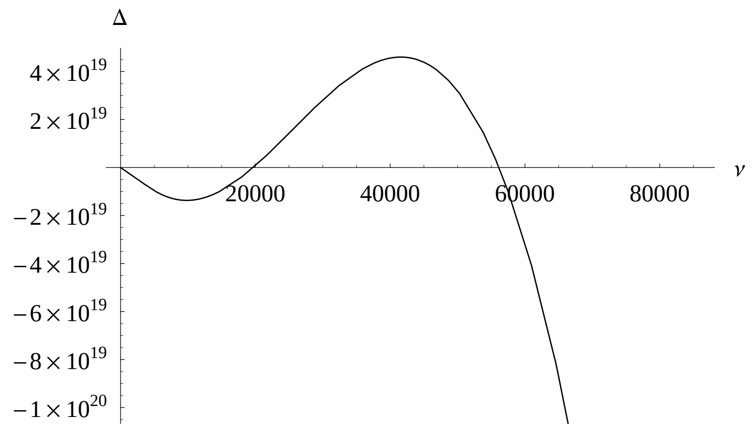

Figure 1: Discriminant (3.15) plotted as a function of volume for the set of parameter values chosen above. This plot demonstrates the excluded volume range where $\Delta < 0$. The discriminant eventually again turns positive as we go to extremely large values of $\mathcal{V}$ which are not included in this plot.

where the discriminant Δ is given by

$$\Delta = 32\chi(2\pi)^{13}M^2 n_7\mathcal{V}^{5/3} + 9\left(n_{D7}\mathcal{V}^{5/3} - N_7\mathcal{V}\right)^2 . \tag{3.15}$$

Since we are considering the case when $\chi < 0$, the above solution can yield an imaginary part when the discriminant turns negative. This happens for small volumes since the first term in (3.15) always dominates at the start. It also takes place in the range of volumes where the terms in the brackets are nearly cancelled. Keeping these restrictions in mind, we can plug (3.14) into (3.13) to obtain the potential as a function of volume. In order to demonstrate how the available parameters can be tuned to give a de Sitter vacuum located at large volume with the string coupling stabilized at a small value we construct a simplified toy model where we set $N_7 = n_7^3 + n_{D7}^3$ and choose $\chi = -4$, $M = 3$, $n_7 = 1$ and $n_{D7} = 73$. The discriminant plotted in figure 1 as a function of volume yields the volume range where the solution for the string coupling (3.14) is real. Thus, we can plug the string coupling (3.14) into the total effective potential (3.13) and plot it as a function of volume in the allowed range figure 2.

We find that the volume is stabilized at a large value $\mathcal{V}_{min} \sim 3 \times 10^4(\alpha')^3$ where the potential has a de Sitter minimum $V_{dS} \sim 2.4 \times 10^{-12}$. In order to find the stabilized value for g_s we plug $\mathcal{V}_{min}$ into (3.14) and find that the string coupling is fixed at a value $g_s \sim 5 \times 10^{-3}$. The string coupling in (3.14) plotted as a function of volume for the same range where $\Delta > 0$ is given in figure 3. This apparent restriction on the allowed values of $\mathcal{V}$ in the plots above is an artifact created by the method which we used to solve for the minimum. Of course, when we plot the effective potential (3.13) in 3D as a function of $\mathcal{V}$ and g_s, they can vary over all positive values figure 4. The values $\mathcal{V}_{min}$ and g_s obtained for our toy model satisfy both consistency checks. First, we demonstrated that the volume modulus can be stabilized at a large value and our reliance on the BBHL result that does

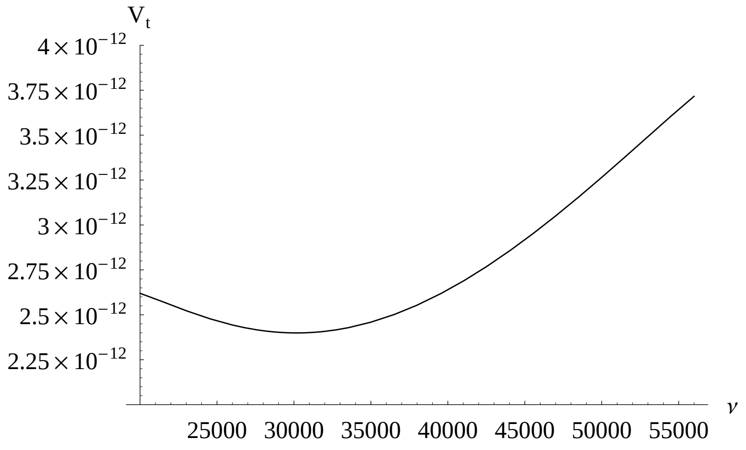

Figure 2: Total effective potential plotted for the volume range where the discriminant $\Delta > 0$. For our particular choice of flux and brane numbers, the volume is stabilized at a large value $\mathcal{V}_{min} \sim 3 \times 10^4 (\alpha')^3$ and the minimum is de Sitter, given by $V_{dS} \sim 2.4 \times 10^{-12}$.

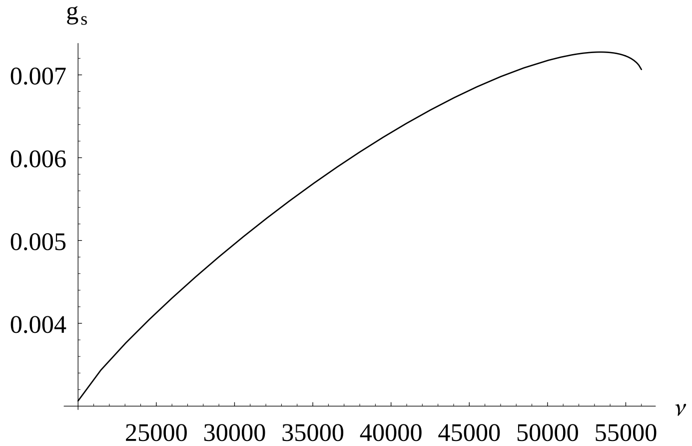

Figure 3: String coupling as a function of stabilized volume plotted in the range where $\Delta > 0$.

not take into account the warp factor is valid. Second, we showed that the string coupling can be stabilized at a small enough value. Recall that the non-perturbative corrections to the tree-level superpotential of the type (2.15) are highly suppressed in the weak coupling limit and therefore we can safely ignore them in our construction. To make sense of the stabilized volume dependence on the number of integer fluxes, recall that the three-cycle at the tip of the conifold whose size is given by the minimal warp factor in (2.14) tries to expand to minimize the energy density created by M units of RR flux $F_{(3)}$. Thus, as we pump the energy into the three-cycle at the tip by increasing the integer M the overall volume also grows. Similarly to [28], supersymmetry is broken at the Kaluza-Klein scale. Serving the role of supersymmetry breaking order parameter is the gravitino mass given by

$$m_{3/2} \sim e^{K_0/2}|W| \sim (2\pi)^5 M \left(\frac{g_s^2}{\mathcal{V}_{min}}\right) \sim 2.4 \times 10^{-5} M_p \sim 6 \times 10^{13} \text{ GeV}. \qquad (3.16)$$

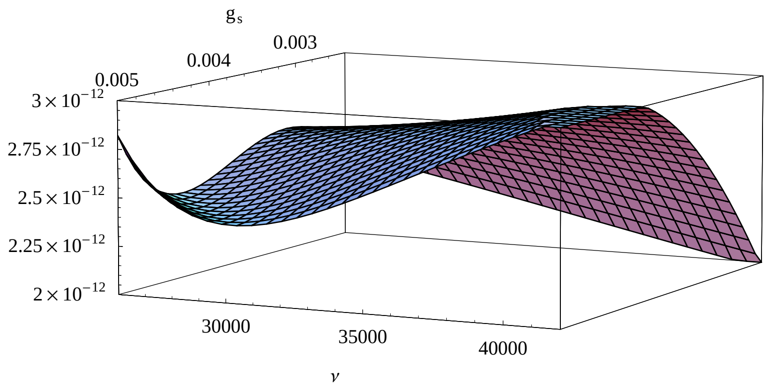

Figure 4: Effective potential (3.13) plotted as a function of string coupling and volume. The values that the volume and the string coupling can take are not restricted. The metastable de Sitter vacuum can be tuned to a desired value by discretely varying flux, topological and brane numbers.

At such high SUSY breaking scale the flexibility provided by the choice of flux, topology and brane numbers gives us the necessary theoretical control over the perturbative expansion in the low-energy effective field theory. In other words, we can tune the volume to a large value to minimize energy densities and stabilize the coupling at a very small value which gives us control over various higher order loop corrections. Our construction can also include mobile D3 and $\overline{D3}$-branes. Due to their interaction with the background fluxes, the $\overline{D3}$-branes naturally move to the highly warped region at the bottom of the KS throat and the corresponding term in the effective potential would have the same scalings as the last term in (3.13). In this type of a brane-world model, the hierarchy of scales is then generated by the warp factor [1, 2, 11] as opposed to the low-energy supersymmetry.[7]

Our mechanism of volume stabilization has an important application for brane-antibrane inflationary models. The authors of [32] consider a particular brane-antibane inflationary scenario in a warped compactification geometry inspired by the KKLT model [17]. In this case the warping provides for a very flat inflaton potential naturally suitable for inflation. However, as the authors in [32] point out, the superpotential volume stabilization based on [17] creates a new problem. The D3-brane moduli modify the definition of the imaginary part of the Kähler modulus ρ in the following way:

$$2\hat{\mathcal{V}}^{2/3} = -i(\rho - \bar{\rho}) - k(\phi, \bar{\phi}) . \tag{3.17}$$

Since ρ is a superfield, volume stabilization through non-perturbative corrections to the superpotential fixes $\rho = \rho_0$ but not the volume modulus $\mathcal{V}$. Since the D3-brane moduli $\phi, \bar{\phi}$ play the role of the inflaton field it is important that the inflaton mass stays extremely small to allow for inflation with enough e-foldings. Since in this case the volume of the Calabi-Yau is not fixed directly the terms in the effective potential, when expanded around ρ_0, result in a mass term for the inflaton. This yields a slow-roll parameter $\eta = 2/3$ which is too large for inflation to continue. This problem can be alleviated by including ϕ dependence

[7]See [52, 53]for some models with high SUSY breaking scale.

into the superpotential and tuning the new terms that appear to cancel the mass term. This solution does not seem very satisfactory as it involves fine tuning to one percent level. On the other hand, as discussed in [32], direct volume stabilization mechanism via stringy corrections to the Kähler potential demonstrated in this section, allows to overcome this major obstacle. In this way the volume modulus is stabilized directly so there is no large inflaton mass term generated by the modification of the potential (3.17) due to the D3-brane moduli and the brane-antibrane inflationary model in [32] can be realized without the need of fine tuning.

4. Conclusions

In this work we demonstrated that volume stabilization in a type IIB warped Calabi-Yau compactification with non-trivial NS and RR fluxes can be achieved perturbatively via a combination of stringy α' corrections to the Kähler potential derived in [27] and the contributions from additional 7-brane-antibrane pairs. The volume modulus as well as the string coupling are stabilized at the minimum of the corresponding effective potential provided the Euler characteristic of the Calabi-Yau threefold is negative. By varying the integer parameters such as the flux numbers, the Euler characteristic and the number of brane-antibrane pairs we obtain a discretuum of non-supersymmetric AdS_4, Minkowski and metastable dS_4 vacua in the 4d low-energy effective field theory. Similarly to [28], in our compactification scheme supersymmetry is broken by the fluxes at the Kaluza-Klein scale. In a toy example, we demonstrated that by tuning the flux, topology and brane numbers, the string coupling can be stabilized at a small enough value thus providing a handle on the effects from higher order terms appearing in the loop expansion. Likewise, the stabilized volume can be made large enough to ensure that energy densities stay small. In this way, theoretical control over the perturbative expansion in the low-energy effective field theory, necessary in the absence of low-energy supersymmetry is achieved. Together with [28], where moduli stabilization for a compactification on a product of Riemann surfaces was achieved perturbatively, our construction featuring a type IIB warped compactification on a Calabi-Yau presents another alternative to the KKLT scenario [17]. Apart from being very simple, the volume stabilization mechanism suggested here completely eliminates the large inflaton mass problem in the brane-antibrane inflationary model in warped geometry [32], which makes it even more attractive.

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